A Comprehensive Analysis of the Conduction-Dominated Transient Contact Melting on an Isothermal Surface

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(Received September 26, 1997)

This paper focuses on the development of a comprehensive analytical model for the conduction-dominated transient contact melting occurring on an isothermal surface, in which emphasis is placed on the treatment of an arbitrary strength of the external force. For a small Stefan number, the proposed model agrees reasonably not only with the exact solution for non-contact melting but also with the existing numerical data for close-contact melting. Normalization of the model equations with reference to the steady solution makes it possible to pick up a single consolidated parameter G, by which the constrained melting processes can be effectively classified into three regimes : non-contact, intermediate and close-contact. Taking advantage of the approximate analytical solution available for close-contact melting, the value of demarcation between the regimes of intermediate and close-contact melting is found to be $G=10^3$. It is also revealed for the first time that in the intermediate regime the contact melting system approaches the steady state passing through a damped oscillation.

Key Words: Contact Melting, Transient Process, Consolidated Parameter, Oscillatory Behaviors

Nomenclature

A : Contact area
c : Specific heat
erf : Error function
F : External force normal to the surface
$\tilde{F}$: Dimensionless external force, $FL/(\mu\alpha)$
G : Consolidated parameter, Eq. (27)
h_{sf} : Latent heat of fusion
k : Thermal conductivity
L : Contact length
M : Mass of the phase change material
$\tilde{M}$: Dimensionless mass, $M/(\rho L^2)$
m : Summation index
P : Pressure
Pr : Prandtl number, $\mu c/k$
Ste : Stefan number, $c\Delta T/h_{sf}$
T : Temperature
T_m : Melting point of the phase change material
ΔT : Temperature difference
t : Time
$\tilde{t}$: Dimensionless time, $t\alpha/L^2$
$\hat{t}$: Normalized time, Eq. (26)
$\Delta\tilde{t}$: Dimensionless time step
u, v : Velocity components, Fig. 1
V : Solid descending velocity
$\tilde{V}$: Dimensionless solid descending velocity, VL/α
$\hat{V}$: Normalized solid descending velocity, $\tilde{V}/\tilde{V}_s$
$\Delta\hat{V}$: Deviation of the normalized velocity
x, y : Cartesian coordinates, Fig. 1

Greek characters

α : Thermal diffusion coefficient, $k/(\rho c)$
δ : Film thickness
$\tilde{\delta}$: Dimensionless film thickness, δ/L
$\hat{\delta}$: Normalized film thickness, $\tilde{\delta}/\tilde{\delta}_s$
$\Delta\hat{\delta}$: Deviation of the normalized thickness
ζ : Dimensionless y, y/δ
λ : Coefficient, Eqs. (17) and (19)
μ : Viscosity
ρ : Density
θ : Dimensionless temperature,

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$(T-T_m)/\Delta T$

ϕ : Dimensionless interfacial temperature gradient, Eq. (16)

Superscript

o : The previous time step

Subscripts

app : Approximate solution

N : Neumann solution

s : Steady state

1. Introduction

Contact melting phenomena occur in numerous natural and technological processes (Moallemi et al., 1986). From the engineering viewpoint, the primary interests in contact melting are placed on high heat transfer rate and/or low friction between two solid bodies concerned. In this connection, a great number of researches have been conducted for diverse geometric configurations, heating modes, contact conditions and types of relative motion between the mating solid surfaces, which have been systematically reviewed by Bejan (1994).

Focusing our attention on close-contact melting among the topics encompassed in the review, it is observed that most of the previous analytical studies have dealt with the quasi-steady process only. That is, the transient process undergone either from the initial direct contact to the steady melting or from one to another steady states of contact melting caused by a sudden change in the melting conditions has been excluded. This is possibly because the unsteady effect is small enough to be neglected, or because inclusion of it no longer allows the problem to be handled analytically, although the rationale for exclusion of it has never been reported yet. However, considering that thorough understanding of time-dependent characteristics such as the duration of transition and variation patterns of the system variables is a prerequisite for justifying the quasi-steady assumption, the transient process needs to be addressed rigorously. Of course, the nature of transition itself is worthy of study in this area.

Hong and Saito (1993) appear to be the first to investigate the initial transient process as a distinct mechanism during close-contact melting occurring between a phase change material and a heated flat surface. They numerically solved full-scale governing equations for the unsteady fluid flow and heat transfer through the liquid film from the onset to the steady state of melting. The simulated results are encouraging in that time evolutions of the system variables have been explicitly illustrated. Despite such an achievement, this study still suffers from a shortage of effort for modeling the fundamental features of the transient process. In fact, their target was primarily directed at developing a numerical method for the moving boundary problem constrained by an externally imposed contact force. Most recently, a remarkable progress in analytical modeling of close-contact melting has been accomplished by Yoo (1997). Based on a set of simplified model equations, he succeeded in deriving an approximate analytical solution for the initial transient process of conduction-controlled close-contact melting. The solution, though it is compactly expressed, seems to be capable of resolving almost all the effects of pertinent factors. Strictly speaking, however, this solution is valid only when the inertial force due to the solid descending motion is negligible compared to the external force exerted on the solid block.

If the melting processes were tentatively classified by the strength of contact force acting between a phase change material and a heating surface, three generic regimes could emerge : non-contact, intermediate and close-contact. The first one, which is free completely from contact and inherently transient in nature, corresponds to the well-known Stefan problem, and the last one, in which the contact force is strong enough, has been surveyed above. In practice, the restriction imposed on Yoo's solution (1997) is equivalent to the condition of close-contact melting. The second one, where the contact force is relatively weak, may also be called contact melting in a rough sense. Contact melting under a micro-gravity or controlled external force environment belongs to this category. In this regime the inertial force would be comparable to the external force. Nevertheless, neither an attempt to categorize the melting processes in such a manner, nor an investigation on the intermediate regime has been published.

As the first step to approach these challenging issues, the present study is intended to establish a characteristic parameter for reasonable classification and to analyze the melting behaviors comprehensively including the intermediate regime. To these ends, an analytical model which can cover all the regimes of melting needs to be developed. Some simplifying assumptions are introduced to render the model equations well-posed within the extent of the fundamental features of contact melting being kept. An appropriate normalization of the model equations facilitates to reach the present purposes.

2. Modeling

The physical system considered in the present study is a simple but representative configuration of contact melting (Bejan, 1995), as depicted schematically in Fig. 1. This system is selected not only because it admits compact formulations, but because numerical data for comparison are also available (Hong and Saito, 1993). Initially a block of phase change material kept at the melting point, T_m, lies on a flat surface, contacting directly with each other over the length, L, by a prescribed normal force, F. Contact melting by its own weight of the solid block, i.e. by the gravity, treated in the previous studies (Moallemi et al., 1986 ; Hong and Saito, 1993 ; Yoo, 1997) is a special case of this model. At $t=0$ the surface starts to be heated isothermally at a temperature, $T_m+\Delta T$, which induces the block to melt. At the same time thin liquid film forms between two solid bodies by melting, and the melt flows toward the ends by the descending motion of the solid block, eventually being squeezed out through the end openings. Both the solid descending velocity and the liquid film thickness vary with time until the steady state is attained, and remain constant thereafter, as far as the contact force acts. At the steady state the solid descending velocity coincides with the melting rate at the solid-liquid interface. The present work traces such a transient process. Similar phenomena may also take place if the melting conditions are changed suddenly during a steady contact melting. Noting that the only difference lies in the initial condition, the same framework of analysis can be applied for them.

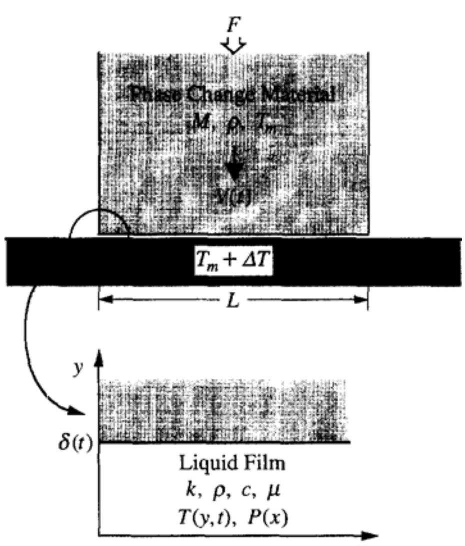

Fig. 1 Schematic of the contact melting system considered in the present study.

For simplicity of modelling, the following assumptions are introduced :

(a) Heat transfer across the liquid film is dominated by conduction. It is an established fact that the assumption holds for small Stefan numbers (Hong and Saito, 1993 ; Bejan, 1994). Yoo et al. (1997) have recently shown that the feasible range was about Ste<0.1.

(b) Melting occurs one-dimensionally in the direction normal (transverse) to the surface, i.e. heat transfer in the longitudinal direction is absent. This is the case for conduction-dominated close-contact melting (Bejan, 1989). Even in the presence of convection in the film, the assumption has already turned out to be valid (Hong and Saito, 1993 ; Yoo et al., 1997).

(c) The density of the phase change material is constant. Although the solid-liquid density difference affects the melting behaviors, it does not alter them substantially (Bejan, 1992 ; Yoo, 1997). Moreover, the effect can be incorporated in the present model without special difficulties if necessary.

(d) The mass of the solid block is invariant throughout the transient process. In the normal contact melting the transient process is very short in comparison to the quasi-steady process where most of the mass melts. Accordingly, the effect of mass variation during the transient process can be neglected (Hong and Saito, 1993).

The target unknowns of the system, time evolutions of the solid descending velocity, $V(t)$, and the liquid film thickness, $\delta(t)$, are determined from the dynamic force balance principle and the energy balance at the solid-liquid interface. First, the vertical force balance relates the inertia of the descending solid with the pressure force developed in the film and the external force exerted on the block, i.e.

$$M\frac{dV}{dt}=F-\int_A PdA \quad (1)$$

for which the assumption (d) has been applied. Note here that the inertia term on the LHS of Eq. (1) has been neglected in the previous analyses of close-contact melting where the external force predominates over it (Moallemi et al., 1986 ; Yoo, 1997). In contrast, the term is retained in this work to include the aforementioned intermediate regime. Next, the interfacial energy balance under the assumption (b) is expressed as

$$-k\frac{\partial T}{\partial y}\bigg|_{y=\delta}=\rho h_{sf}\left(V+\frac{d\delta}{dt}\right) \quad (2)$$

where the sum of the solid descending velocity and the growth rate of film thickness in the parenthesis on the RHS represents an instantaneous melting rate. Thus we have derived a set of simultaneous ordinary differential equations for the system variables, $V(t)$ and $\delta(t)$, subject to the initial conditions,

$$V(0)=\delta(0)=0 \quad (3)$$

It should be mentioned that Eq. (3) is valid only when the assumption (c) holds. Otherwise, $V(0)$ could depend on the solid-liquid density ratio (Yoo, 1997).

For the closure of model equations, the pressure distribution in the liquid film and the temperature gradient at the melting front appeared in Eqs. (1) and (2), respectively, should be specified in terms of the system variables. This can be accomplished by solving the continuity, momentum and energy equations in the liquid film. Owing to the assumption (c), the continuity equation is simply

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (4)$$

According to the classical theory of lubrication (Batchelor, 1967 ; Bejan, 1989), the liquid inertia and the pressure variation in the transverse direction are negligible, so that the momentum equation is simplified as

$$\frac{dP}{dx}=\mu\frac{\partial^2 u}{\partial y^2} \quad (5)$$

Upon applying the assumptions (a) and (b), the energy equation reduces to

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (6)$$

In order to comprehend all the regimes of melting, the unsteady term is retained in the energy equation. If we are interested in close-contact melting only, the term can be legitimately dropped even for the transient analysis (Yoo, 1997).

Since the simplified continuity and momentum equations are identical with those for the steady analysis (Bejan, 1995 ; Yoo et al., 1997), the solution procedure for fluid mechanics part is not repeated here, but the result only is presented. A parabolic velocity profile across the film, $u(y)$, is obtained from Eq. (5) and the no-slip boundary conditions, $u(0)=u(\delta)=0$. This profile together with the impermeable bottom and blowing interfacial conditions for the transversal velocity component, $v(0)=0$ and $v(\delta)=-V$, enables Eq. (4) to be integrated from $y=0$ to $y=\delta$. Thus we have the following differential equation for the pressure distribution along the film :

$$\frac{d^2P}{dx^2}=-\frac{12\mu V}{\delta^3} \quad (7)$$

Integrating Eq. (7) twice with the end conditions, $P(0)=P(L)=0$, substituting the pressure distribution into Eq. (1), and performing the integration finally yield

$$M\frac{dV}{dt}=F-\mu V\left(\frac{L}{\delta}\right)^3 \tag{8}$$

The energy equation, Eq. (6), subject to the initial and boundary conditions,

$$T(y,0)=T_m \tag{9}$$

$$T(0,t)=T_m+\Delta T\ ;\ T(\delta,t)=T_m \tag{10}$$

respectively, does not allow an exact solution not only because one of the boundaries, i.e. the solid-liquid interface, is unknown *a priori*, but because it is also coupled with the force balance. This obstacle can be circumvented by invoking an additional assumption that the instantaneous velocity of the interface is negligible, i.e. the growth rate of the film is very low, which will be justified later. Then there exists an approximate solution for this problem (Carslaw and Jaeger, 1959 ; Yoo, 1997),

$$\theta=(1-\zeta)-\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}e^{-am^2\pi^2t/\delta^2}\sin(m\pi\zeta) \tag{11}$$

Applying the interfacial temperature gradient determined from Eq. (11) into Eq. (2), we have the following energy balance equation expressed in terms of the system variables

$$\rho h_{sf}\left(V+\frac{d\delta}{dt}\right)=\frac{k\Delta T}{\delta}\left[1+2\sum_{m=1}^{\infty}(-1)^m e^{-am^2\pi^2t/\delta^2}\right] \tag{12}$$

3. Validation

So far, simplified but completed model equations, Eqs. (8) and (12), describing the transient process of contact melting have been developed. Now they are converted into convenient forms to be validated. Referring to the definitions given in Nomenclature and rearranging terms, we obtain the dimensionless system of model equations and boundary conditions,

$$\frac{d\tilde{V}}{d\tilde{t}}=(\mathrm{Pr}\tilde{M}^{-1})(\tilde{F}-\tilde{V}\tilde{\delta}^{-3}) \tag{13}$$

$$\frac{d\tilde{\delta}}{d\tilde{t}}=(\mathrm{Ste}\phi)\,\tilde{\delta}^{-1}-\tilde{V} \tag{14}$$

$$\tilde{V}(0)=\tilde{\delta}(0)=0 \tag{15}$$

The abbreviated term ϕ in Eq. (14), which is defined as

$$\phi=\left.\frac{\partial\theta}{\partial\zeta}\right|_{\zeta=1}=1+2\sum_{m=1}^{\infty}(-1)^m e^{-m^2\pi^2\tilde{t}/\tilde{\delta}^2} \tag{16}$$

represents the dimensionless interfacial temperature gradient.

It is important to validate the model equations prior to advancing to the main targets since not a few assumptions have been introduced during the formulation procedure. Two limiting cases, i.e. the regimes of non-contact and close-contact melting, for which the exact solution and numerical data are available, respectively, are considered for verification.

In the non-contact Stefan problem, there is no solid motion ($F=V=0$) so that Eq. (13) is irrelevant to the system behaviors. The solid-liquid interface in this case is free to move, whereas it is constrained by the external force in contact melting. Accordingly, the interfacial velocity that has been neglected to derive Eq. (11) is highest in non-contact melting. It is obvious that an error caused by the assumption should diminish sharply with increasing contact force (F). That is, this limiting case provides an upper bound of the error for contact melting. The well-known Neumann solution indicates that the interface position moves according to $\tilde{\delta}=2\lambda_N\tilde{t}^{1/2}$, where the coefficient λ_N is the solution of the following transcendental equation :

$$\lambda_N e^{\lambda_N^2}\mathrm{erf}(\lambda_N)=\mathrm{Ste}/\pi^{1/2} \tag{17}$$

On the other hand, the approximate interfacial energy balance, Eq. (14), is rearranged as

$$\frac{1}{2}\frac{d\tilde{\delta}^2}{d\tilde{t}}=\mathrm{Ste}\left[1+2\sum_{m=1}^{\infty}(-1)^m e^{-m^2\pi^2\tilde{t}/\tilde{\delta}^2}\right] \tag{18}$$

By taking $\tilde{\delta}=2\lambda_{app}\tilde{t}^{1/2}$, Eq. (18) reduces to an algebraic equation for the coefficient λ_{app}

$$\lambda_{app}^2=\mathrm{Ste}\left[1+2\sum_{m=1}^{\infty}(-1)^m e^{-m^2\pi^2/(4\lambda_{app}^2)}\right] \tag{19}$$

Then the dimensionless interfacial temperature gradient ϕ, which directly affects the system behaviors in Eq. (14), can be expressed in terms

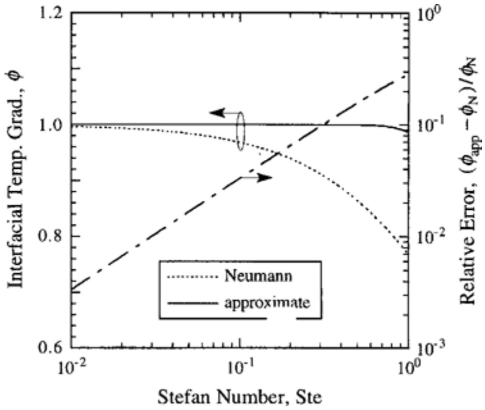

Fig. 2 Comparison of the interfacial temperature gradient between the approximate and exact solutions for the Stefan problem as a function of the Stefan number.

of the coefficient λ simply as $\phi_N=2\lambda_N^2/\mathrm{Ste}$ and $\phi_{app}=2\lambda_{app}^2/\mathrm{Ste}$, respectively. Variations of ϕ_N, ϕ_{app}, and the relative error between them with respect to the Stefan number are shown in Fig. 2. As Ste increases, ϕ_N gradually decreases, while ϕ_{app} being kept nearly constant. Hence the discrepancy between them increases. Fortunately, the relative error for the conduction-dominated range, i.e. Ste$<$0.1, remains within a tolerable limit (about 3.3 % or less). In view of the above discussion on non-contacting melting, the fundamental features of contact melting seem to be hardly affected by the assumption. An another meaningful result is that $\phi\cong 1$ for small Stes, which closely resembles the approximation of linear temperature profile across the liquid layer (Carslaw and Jaeger, 1959). This leads to the fact that ϕ in conduction-dominated contact melting depends neither on time nor on Ste.

In the regime of close-contact melting, Hong and Saito (1993) presented two sets of simulated results for Ste$=$0.01266 and 1.266, respectively. Only the former is adopted here for comparison since the latter, in which the convection effect across the film was reported to be significant, is apparently out of the present scope. Listed in Table 1 are the specific melting conditions under the gravitational field in the present notation, where the last column is appended for later use. Meanwhile, the dimensionless model equation

Table 1 Conditions of close-contact melting used for validation.

Parameter	Ste	Pr	$\tilde{M}$	$\tilde{F}$	G
Value	0.01266	13.44	0.5	1.643×10^{11}	4.030×10^{6}

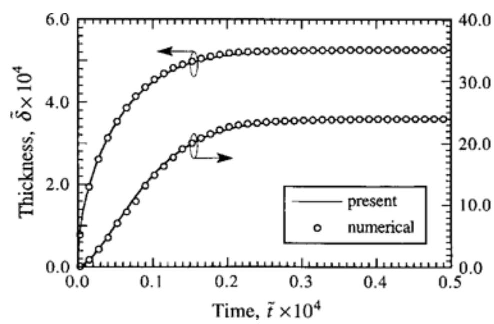

Fig. 3 Comparison of time evolutions of the liquid film thickness and solid descending velocity between the present results and available numerical data under the conditions of Table 1.

system, Eqs. (13) ~ (15), should be solved by numerical means due to the inherent nonlinearity. Conventional ordinary differential equation solvers such as the predictor-corrector or Runger-Kutta method can not be employed for this work since the model equations involve the terms which are indefinite at $\tilde{t}=0$ (Ferziger, 1981). Therefore, the backward-difference discretization is invoked, which results in the following simultaneous nonlinear equations :

$$(\mathrm{Pr}\tilde{M}^{-1})(\tilde{F}-\tilde{V}\tilde{\delta}^{-3})\Delta\tilde{t}-(\tilde{V}-\tilde{V}^{o})=0 \quad (20)$$

$$[(\mathrm{Ste}\phi)\tilde{\delta}^{-1}-\tilde{V}]\Delta\tilde{t}-(\tilde{\delta}-\tilde{\delta}^{o})=0 \quad (21)$$

where superscript o denotes the value at the previous time step. Using the Newton-Raphson method, Eqs. (20) and (21) converge stably and efficiently. The calculated results, i.e. time evolutions of the solid descending velocity $\tilde{V}$ and film thickness $\tilde{\delta}$, are compared with those by the simulation in Fig. 3. Two set of results agree excellently with each other at all times. This fact together with the foregoing discussion on the Stefan problem seems to suffice to validate the present model.

4. Discussion

4.1 Normalization

The nondimensionalized model equations, Eqs. (13) and (14), are cast in inconvenient forms to capture the fundamental features of contact melting, although they are ready to be solved and compared. The quantities of interest such as time evolutions of the solid descending velocity and the film thickness should be repeatedly calculated for every combination of four dimensionless parameters, Ste, Pr, $\tilde{F}$ and $\tilde{M}$. In addition, a parameter for classifying the regime of melting processes is still uncertain. These issues can be judiciously overcome by normalizing the model equations with reference to the steady solution (Yoo, 1997).

Since at the steady state all time derivative terms vanish, and $\phi=1$ in Eqs. (13) and (14), the steady solution is readily obtained as

$$\tilde{V}_s=(\mathrm{Ste}^3\tilde{F})^{1/4} \tag{22}$$

$$\tilde{\delta}_s=(\mathrm{Ste}\tilde{F}^{-1})^{1/4} \tag{23}$$

These are consistent with the previous steady analysis (Bejan, 1995 ; Yoo et al., 1997). Applying Eqs. (22) and (23) into Eqs. (13) and (14), we obtain the following normalized equations :

$$\frac{d\hat{V}}{d\hat{t}}=G(1-\hat{V}\hat{\delta}^{-3}) \tag{24}$$

$$\frac{d\hat{\delta}}{d\hat{t}}=\phi\hat{\delta}^{-1}-\hat{V} \tag{25}$$

where the normalized time $\hat{t}$ and a new parameter G are defined, respectively, as

$$\hat{t}=\tilde{t}\,\tilde{V}_s/\tilde{\delta}_s \tag{26}$$

$$G=\mathrm{Pr}\tilde{M}^{-1}\mathrm{Ste}^{-5/4}\tilde{F}^{1/4} \tag{27}$$

The term ϕ can also be expressed as a function of the normalized quantities, i.e.

$$\phi=1+2\sum_{m=1}^{\infty}(-1)^m e^{-m^2\pi^2\hat{t}/(\mathrm{Ste}\hat{\delta}^2)} \tag{28}$$

Although it has already been verified that $\phi=1$, the term ϕ is still retained in the model equations. If a better approximate solution instead of Eq. (11) could be obtained, the framework of this analysis would be used simply by replacing the term.

In a strict sense, the steady solution, Eqs. (22) and (23), is meaningful only in the regime of close-contact melting. In the non-contact and intermediate regimes the melting process may not attain the steady state. However, the steady solution in those regimes can be regarded simply as a certain reference quantity apart from its physical meaning since the model equations normalized by it describe the system characteristics more effectively. This argument is readily substantiated by the fact that they apparently involve only two parameters, G and Ste, in contrast to four in the dimensionless forms. Furthermore, $\phi=1$ all the time for Ste<0.1 as discussed, so that the melting behaviors are actually independent of Ste. Thus the normalized equations are characterized by a single parameter. Note here that the parameter G comprises all the melting conditions (in this sense it is termed *the consolidated parameter* hereafter). The consolidated parameter can take the place of the tentatively used contact force, and allows a more reasonable classification of the melting regimes. Three regimes proposed earlier correspond to the cases of $G\rightarrow 0$, the intermediate range of G and $G\rightarrow\infty$, respectively, each of which is discussed below.

4.2 Non-contact melting

The regime of non-contact melting, i.e. $G\rightarrow 0$, physically comes from the case of $\tilde{F}\rightarrow 0$ and/or $\tilde{M}\rightarrow\infty$ since Pr and Ste are finite (see Eq. (27)). In this regime Eq. (24) degenerates to $d\hat{V}/d\hat{t}=0$, which yields $\hat{V}=0$ on applying the initial condition, $\hat{V}(0)=0$. Then Eq. (25) reduces to Eq. (18). The previous chapter appears to suffice for understanding of pertinent phenomena.

4.3 Close-contact melting

The other limiting case, i.e. $G\rightarrow\infty$, which is materialized when $\tilde{F}$ is large enough, corresponds to the classical close-contact melting. The case of $\tilde{M}\rightarrow 0$ should be precluded in the light of the assumption (d) as well as the physical reality. Contact melting due to the normal gravity belongs to this regime (see the value in the last column of Table 1).

As described earlier, Yoo (1997) has obtained an approximate analytical solution for the transient process of close-contact melting by neglecting the inertial force due to the solid descending motion in comparison to the external force. The solution procedure is briefly regenerated here to link it to the discussion on the feasible range of the consolidated parameter. Since the normalized solid descending acceleration, $d\hat{V}/d\hat{t}$, is finite and $G\rightarrow\infty$, Eq. (24) is simplified as

$$\frac{1}{G}\frac{d\hat{V}}{d\hat{t}}=1-\hat{V}\hat{\delta}^{-3}\approx 0 \tag{29}$$

Applying this result, i.e. $\hat{V}\approx\hat{\delta}^3$, and $\phi=1$ into Eq. (25), the following ordinary differential equation for $\hat{\delta}(\hat{t})$ is obtained :

$$\frac{d\hat{\delta}^2}{d\hat{t}}=2(1-\hat{\delta}^4) \tag{30}$$

Equation (30) subject to the initial condition, $\hat{\delta}(0)=0$, is solved to yield

$$\hat{\delta}(\hat{t})=\tanh^{1/2}(2\hat{t}) \tag{31}$$

which together with Eq. (29) leads to

$$\hat{V}(\hat{t})=\tanh^{3/2}(2\hat{t}) \tag{32}$$

Figure 4 depicts typical results of the approximate solution, Eqs. (31) and (32). The curves explicitly delineate time evolutions of the system variables from the beginning to the steady state of close-contact melting. Since validation and other aspects of this solution set have been thoroughly discussed by Yoo (1997), they are not repeated here.

The approximate solution is valid for $G\rightarrow\infty$ and Ste<0.1. The feasible range of the consolidated parameter, i.e. the lower bound of G, can be specified through comparison between the approximate and exact (numerical) solutions of the model equations. Let the deviations in $\hat{V}$ and $\hat{\delta}$ of the approximate solution from the numerical result be $\Delta\hat{V}$ and $\Delta\hat{\delta}$, respectively. Variations of $\Delta\hat{V}$ and $\Delta\hat{\delta}$ for three values of G are plotted in Fig. 5, where all numerical calculations were performed for Ste$=0.01$ to assure $\phi=1$. Apart from time evolution, both deviations decrease sharply with increasing G to negligible levels when $G=10^3$. This leads to the fact that $G\geq 10^3$ is a roughly estimated feasible range of G for the approximate solution to be valid (or for maintaining close-contact melting). In this regard, the value of G listed in Table 1 falls safely within the feasible range as noted earlier. Worthy of remark in the figure are the facts that $|\Delta\hat{V}|$ is greater than $|\Delta\hat{\delta}|$, and that the maxima of them emerge at different times as clearly observable in the case of $G=10$. The former is due to that the neglected term in Eq. (29), i.e. $d\hat{V}/d\hat{t}$, affects $\hat{V}$ more seriously than $\hat{\delta}$. On the other hand, the latter seems to originate from a certain phase difference between $\hat{\delta}$ and $\hat{V}$, though it is obscure at this stage. In addition, $\Delta\hat{V}$ and $\Delta\hat{\delta}$ for $G=10$ and 10^2 vary oscillatorily with time, whereas those for $G=10^3$ asymptotically decrease to zero after a small initial increase. Increases in deviations with decreasing G are evidently caused by that the approximation, Eq. (29), is violated for small Gs.

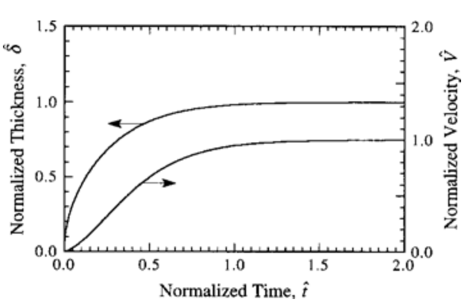

Fig. 4 Time evolutions of the normalized liquid film thickness and solid descending velocity by the approximate analytical solution for close-contact melting.

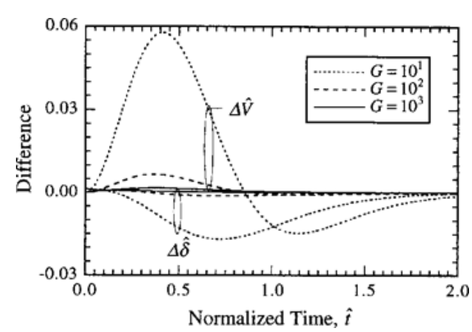

Fig. 5 Effect of the consolidated parameter on timewise deviations in the normalized liquid film thickness and solid descending velocity between the approximate analytical and numerical solutions for Ste$=0.01$.

In order to support the above arguments, variations of numerically calculated $d\hat{V}/d\hat{t}/G$ for each G of Fig. 5 are depicted in Fig. 6. Note that the term $d\hat{V}/d\hat{t}/G$ physically represents the ratio of the inertia to external force. As can be expected, it increases remarkably as G decreases, while keeping similar variation patterns one another. It is very suggestive that time when the acceleration $d\hat{V}/d\hat{t}$ is maximum (Fig. 6) is followed by that of $|\Delta\hat{V}|$ (Fig. 5).

4.4 The intermediate regime

The intermediate range of the consolidated parameter, approximately $0<G<10^3$ based on the foregoing discussion, is of particular interest since it has never been investigated yet. As mentioned earlier, this regime encompasses contact melting under a microgravity or external force controlled by a spring. Although the magnitude of the solid descending acceleration, $d\hat{V}/d\hat{t}$, in this regime is smaller than that in close-contact melting, it can not be neglected due to a small G as shown in Fig. 6. Therefore, the normalized model equations, Eqs. (24) and (25), no longer allow additional simplifications, and should be solved by the numerical method presented previously.

From the mathematical viewpoint, the model equations can be considered to describe a nonlinear dynamical system. Supposing the consolidated parameter G represents a certain combined coefficient of stiffness and damping, the limiting cases of G, i.e. $G\rightarrow 0$ and $G\rightarrow\infty$, could be regarded as monotonically diverging and converging systems, respectively, in the light of time evolution of $\hat{\delta}$. It is plausible here to deduce that the system variables behave oscillatorily in the intermediate regime. In fact, such oscillations in both $\hat{\delta}$ and $\hat{V}$ have ever appeared partly, for example, in the case of $G=10$ in Fig. 5. Referring to Fig. 4, the process of close-contact melting seems to attain the steady state near $\hat{t}=1.5$, which is also supported by $d\hat{V}/d\hat{t}\cong 0$ in Fig. 6. Then a negative value of $\Delta\hat{V}$ or $\Delta\hat{\delta}$ at that time for $G=10$ in Fig. 5 implies that the actual

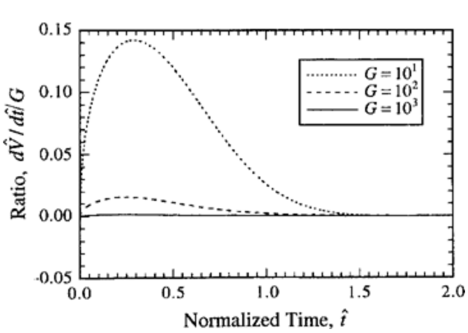

Fig. 6 Effect of the consolidated parameter on timewise variation of the inertia-to-external force ratio for Ste=0.01.

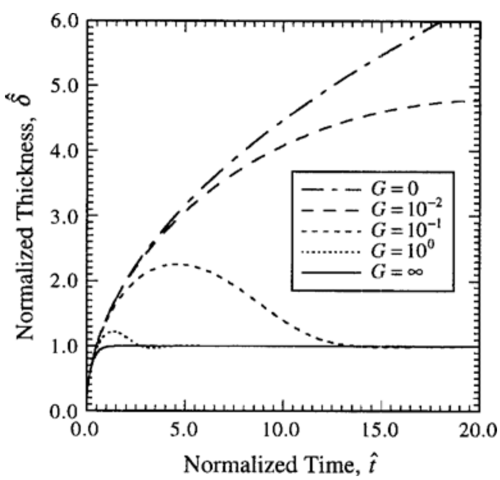

Fig. 7 Time evolutions of the normalized liquid film thickness at different values of the consolidated parameter for Ste=0.01.

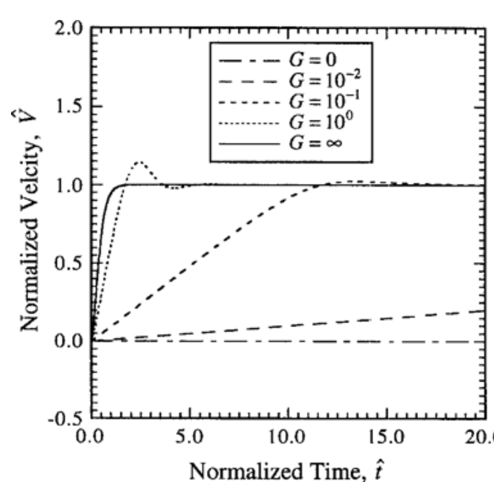

Fig. 8 Time evolutions of the normalized solid descending velocity at different values of the consolidated parameter for Ste=0.01.

(numerically calculated) $\hat{V}$ or $\hat{\delta}$ is already greater than unity, thereby approaching the steady state in a decreasing fashion. Note that in Fig. 4 they vary monotonically with time.

In order to examine the oscillatory behaviors more comprehensively, the numerical results for some selected values of the consolidated parameter are illustrated together with two limiting cases of G in Figs. 7 (for the thickness δ) and 8 (for the velocity $\hat{V}$), for which Ste$=0.01$ was used as before. A pair of figures successfully delineate a global map of transient melting processes. Oscillations both in the film thickness and in the solid descending velocity are grasped explicitly, though they decay very rapidly with time. The amplitude and period of oscillation seem to be reduced as G increases. This may be better understood by matching the external force acting on the solid block, pressure built up in the film and viscosity of the liquid (via Pr) to the excitation force, stiffness and damping coefficients. In this manner, non-contact ($G \to 0$) and close-contact ($G \to \infty$) melting correspond to the undamped and overdamped systems having infinite/zero and zero/infinite periods/frequencies, respectively.

Figure 9 illustrates enlarged and overlapped variation patterns of $\hat{\delta}$ and $\hat{V}$ during contact melting in the intermediate regime ($G=1$ and 2

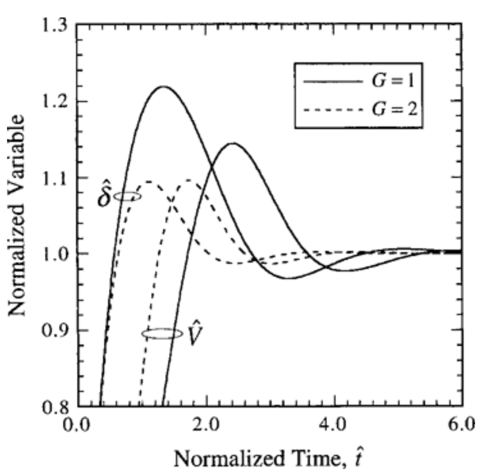

Fig. 9 Oscillatory behaviors of the normalized liquid film thickness and solid descending velocity during the transient process of contact melting in the intermediate regime for Ste $=0.01$.

as representative cases). The oscillatory behaviors shown there can be interpreted physically as follows. At the early stage of melting, the liquid film responds promptly to an almost infinitely high initial heat transfer rate so that it grows sharply, while the solid block moves downward very slowly because of a high film pressure resisting the motion. Since the pressure force decreases as the film thickens (see the second term on RHS of Eq. (8)), the external force begins to prevail in the dynamic force balance. Then the solid descending motion is enhanced, which in turn causes the liquid gap to be thinner, thereby raising the pressure there. The pressure rise yields a reduction of the solid descending velocity. These types of alternating phenomena continue until the steady state. The oscillations are damped rapidly once the film thickness has exceeded a certain level because they were initiated primarily by the high (direct contact) heat transfer rate. In this context, found in Fig. 9 are the facts that there exists a phase difference between $\hat{\delta}$ and $\hat{V}$, and that it depends essentially on G. Here the previous statement on the phase difference associated with Fig. 5 may be justified. Note also that the phase angle between the displacement and velocity in a harmonic system is $\pi/2$.

Finally, it is important to confirm whether or not the foregoing theoretically predicted phenomena actually occur. Considering that the film thickness is very thin, verification by means of an experiment would require the highest degree of care. Rather, a sophisticated simulation might accommodate it.

5. Conclusions

A simple analytical model for the conduction-dominated contact melting occurring between a phase change material kept at its melting point and an isothermally heated flat surface has been proposed, which comprehends every type of the transient process regardless of the strength of contact force acting between two solids. The model equations basically consist of the dynamic force balance on the solid block and the energy balance at the solid-liquid interface, for the closure of which the continuity, momentum and energy equations in the liquid film have been handled analytically.

For the range of small Stefan number, movement of the solid-liquid interface proved to affect the transient temperature profile so slightly as to be neglected within a tolerance even in non-contact melting where the growth rate of the film thickness is highest. In addition, the calculated time evolutions of the system variables agreed excellently with the existing numerical data for close-contact melting . These comparisons for two limiting cases lead to the conclusion that the present model has been developed properly and is capable of predicting the fundamental features of contact melting.

Normalization of the model equations with reference to the steady solution has effectively demarcated the transient melting processes. The normalized equations depend on a single characteristic parameter G which is termed the consolidated parameter after its nature. Hence the contact melting systems can be reasonably classified by this parameter into three regimes: non-contact, intermediate and close-contact. The small extreme of G, i.e. $G \rightarrow 0$, designates non-contact melting, which is identical with the well-known Stefan problem. On the other hand, the opposite extreme of G describes close-contact melting. Based on the approximate analytical solution available in this regime, the feasible range of close-contact melting could be specified as $G \geq 10^3$.

In the intermediate regime ($0 < G < 10^3$), the contact melting system behaves similarly to a nonlinear dynamical system. Both the film thickness and the solid descending velocity oscillatorily approach the steady state, while showing a phase difference between them and being damped rapidly with time. In view of the fact that both the amplitude and period in the film thickness as well as the solid descending velocity decrease with increasing G, it can be deduced that the regimes of non-contacting and close-contact melting correspond to an undamped system with infinite period and an overdamped system with zero period, respectively. However, these theoretically detected oscillatory behaviors in the intermediate regime need appropriate verifications by other means.

Acknowledgement

The author would like to acknowledge the financial support of this work by 1997 Soong Sil University Research Fund.

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